Comments on "Infrared Radiometry of Turbulent Flows"

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IN a very interesting article, Draper[1] reported on an investigation of the infrared fluctuations radiated by the fluid-mechanical turbulence of a hot free jet. The radiation was detected by an infrared radiometer having an optical field that extended entirely across the jet flow and had variable resolution along the flow axis (for a given position downstream of the jet orifice).

Draper's theoretical analysis of the radiometer fluctuation spectrum leads to the result that the radiometer spectrum is proportional to the product of the square of the Fourier transform of the radiometer aperture function and the one-dimensional temperature fluctuation spectrum [Eq. (38)]. When the radiometer aperture width along the flow axis approaches zero as a limit, the radiometer spectrum is then simply proportional to the one-dimensional temperature spectrum (independent of the transverse resolution of the radiometer). This appears to be an improbable result, for it implies that a sensor such as the radiometer, with no resolution in two-dimensions (i.e., no resolution transverse to the jet axis), has the same spectral response as a sensor having point resolution such as a hot wire anemometer. This result is a consequence of Draper's assumption for the form of the spectral signal $T(n, r)$ [Eq. (30)]. The basis for using this equation as a starting point for a theoretical derivation is unclear to this author. An alternate theoretical analysis shall be derived below.

The correlation function of the radiometer temporal fluctuations $R_p(\tau)$ can be expressed (in the notation of Ref. 1) as†

$$R_p(\tau) = \frac{\overline{\Delta P(t)\Delta P(t+\tau)}}{\overline{\Delta P'^2}} = \iiint_V \iiint_{V'} A(x)A(x')A(y)A(y')A(z)A(z') \times \frac{\overline{\Delta T_Q(t)\Delta T_R(t+\tau)}}{\overline{\Delta T'^2}}\, dv dv' \quad (1)$$

After Tatarski,[2] Eq. (1) can be expressed as

$$R_p(\tau) = \iiint_V A(x)A(y)A(z)\; R(x,y,z,\tau)\; dxdydz \quad (2)$$

where

$$R(x, y, z, \tau) = [\overline{\Delta T_Q(t)\Delta T_R(t+\tau)}]/\overline{\Delta T'^2}$$

and when the dimensions of the turbulent medium are much greater than the correlation length. Using Taylor's hypothesis, time is equivalent to x directed displacement, so that Eq. (2) becomes

$$R_p(\tau) = \iiint_V A(x)A(y)A(z)\; R(x - \bar{U}\tau, y, z) dx\, dy\, dz \quad (3)$$

The correlation function is related by a three-dimensional Fourier transform to its power spectral density function in wavenumber space $G(k_x,k_y,k_z)$. Equation (3) then can be rewritten, so that it becomes

$$R_p(\tau) = \iiint_V dxdydz\, A(x)A(y)A(z) \times \iiint_{-\infty}^{\infty} dk_x dk_y dk_z G(k_x,k_y,k_z) \times \exp i[(k_x x + k_y y + k_z z) - k_x \bar{U}\tau] \quad (4)$$

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† Constants of proportionality shall be omitted throughout the text because only normalized spectra shall be considered.

Express the $(y - z)$ plane in polar coordinates. Assuming circular symmetry of the jet so that the radiometer aperture function is a function of radius alone, Eq. (4), after integration over the azimuth angle, becomes

$$R_p(\tau) = \int_{-\infty}^{\infty}\int_{-0}^{\infty} dxdr\; r\, A(x)A(r) \times \iiint_{-\infty}^{\infty} dk_x dk_y dk_z G(k_x, k_y, k_z) \times J_0[r(k_z^2 + k_y^2)^{1/2}] \exp - ik_x\bar{U}\tau \quad (5)$$

where J_0 is the Bessel Function of the first kind of zero order.

Transforming $R_p(\tau)$ into frequency space and integrating over k_x, we obtain

$$E_p(n) = \int_{-\infty}^{\infty}\int_{0}^{\infty} dxdr\; r\, A(r)A(x)\, \frac{1}{\bar{U}} \times \iint_{-\infty}^{\infty} dk_y dk_z\; G\left(\frac{2\pi n}{\bar{U}}, k_y, k_z\right) J_0[r(k_z^2 + k_y^2)^{1/2}] \quad (6)$$

Express the k_y-k_z plane in polar coordinates, so that $k^2 = k_y^2 + k_z^2$, and integrate over the azimuthal angle. Equation (6) becomes, after substituting for the temperature fluctuation spectrum (and assuming that the correlation function of the temperature fluctuations is exponential),

$$E_p(n) = \int_{-\infty}^{\infty}\int_{0}^{\infty} dxdr\; r\, A(r)A(x)\, \frac{1}{\bar{U}} \times \int_0^{\infty} dk\, \frac{k\, J_0(kr)}{[(4\pi^2 n^2/\bar{U}^2) + k^2 + \Lambda^{-2}]^2} \quad (7)$$

The integral over k can be expressed in terms of the modified Bessel Function of the second kind K_1, so that Eq. (7) can be expressed[3]

$$E_p(n) = \int_{-\infty}^{\infty}\int_{0}^{\infty} dxdr\; A(x)A(r)r^2 \times \frac{K_1[r(\Lambda^{-2} + 4\pi^2 n^2/\bar{U}^2)^{1/2}]}{\bar{U}(\Lambda^{-2} + 4\pi^2 n^2/\bar{U}^2)^{1/2}} \quad (8)$$

Equation (8) is the solution we have been seeking.

The limiting solutions of Eq. (8) will now be investigated. In the limit, when the x direction aperture width is zero [so that $A(x)$ can be represented as a delta function] and when

$$A(r) = [\bar{U}(r)]/\bar{U}_m = 1 \qquad \text{independent of } r$$

the radiometric energy spectrum is proportional to n^{-4} for $(4\pi^2 n^2/\bar{U}^2) \gg \Lambda^{-2}$. Alternatively, if the radiometer is assumed to have point resolution transverse to the flow vector, i.e.,

$$A(r) = [\delta(r)]/2\pi r = \text{delta function}$$

the radiometric energy spectrum is proportional to n^{-2} for $(4\pi^2 n^2/\bar{U}^2) \gg \Lambda^{-2}$. It is seen that only in the limit of point resolution does the spectrum approach the hot wire spectrum. To evaluate the predicted radiometric spectrum for the jet of Ref. 1, the experimental parameters can be substituted into Eq. (8), and the integration performed for several frequencies.

References

[1] Draper, J. S., "Infrared radiometry of turbulent flows," AIAA J. **4**, 1597–1603 (1966).

[2] Tatarski, V. I., *Wave Propagation in a Turbulent Medium* (McGraw-Hill Book Company Inc., New York, 1961).

[3] Magnus, W. and Oberhettinger, F., *Formulas and Theories for the Function of Mathematical Physics* (Chelsea Publishing Company, New York, 1943).